NUMBER THEORY.

218. Proposed by ELIJAH SWIFT, University of Vermont.

If p is a prime >3, show that $\sum_{a=1}^{p-1} \frac{1}{a^2} \equiv 0 \pmod{p}$.

II. SOLUTION BY THE PROPOSER.

It may be proved (Bachmann, *Niedere Zahlentheorie*, page 155) that $A_{p-3} = 1 \cdot 2 \cdot 3 \cdots p-3 + 1 \cdot 3 \cdot 4 \cdots p-2 + \cdots$, taking all possible combinations of the numbers 1, 2, $\cdots p-1$, $p-3$ at a time, is divisible by p.

Then

$$A_{p-3} = (p-1)! \sum_{\substack{a=1 \\ b=1}}^{\substack{a=p-1 \\ b=p-1}} \frac{1}{a \cdot b} \qquad a \neq b.$$

Then

$$\Sigma \frac{1}{a \cdot b} \equiv 0 \pmod{p}.$$

This sum may be written

$$\frac{1}{2}\left\{ \sum_{b=1}^{p-1} \frac{1}{1 \cdot b} - \frac{1}{1^2} + \sum_{b=1}^{p-1} \frac{1}{2 \cdot b} - \frac{1}{2^2} + \cdots + \sum_{b=1}^{p-1} \frac{1}{(p-1)b} - \frac{1}{(p-1)^2} \right\}.$$

But

$$\sum_{b=1}^{p-1} \frac{1}{a \cdot b} \equiv \sum_{b=1}^{p-1} \frac{b}{a} \equiv \sum_{b=1}^{p-1} b \equiv 0 \pmod{p}.$$

Hence,

$$\Sigma \frac{1}{ab} \equiv -\sum_{a=1}^{p-1} \frac{1}{a^2} \equiv 0 \pmod{p}.$$

QUESTIONS AND DISCUSSIONS.

EDITED BY U. G. MITCHELL, University of Kansas.

NEW QUESTIONS.

26. Why should not the nomenclature of mathematics be made uniform? For example, why call a circle a *portion of a plane* in elementary geometry and a *curve* in analytic geometry? Why call a sphere a *ball* at one time and a *surface* at another time? And so on through all the configurations of two- and three-dimensional geometry.

REPLIES.

18. In view of the present pressure for saving time and gaining efficiency, what are the most important sources of economy in the mathematical courses of the high school and the first two years in college?

REPLY BY ERNEST W. PONZER, Leland Stanford Junior University.

This question assumes the existence and application of the fundamental principles of efficiency[1] in connection with our college instruction in mathematics. No doubt the editors are aware of the fact that many college professors would hardly make this assumption and that many others would regard the problem involved as quite trivial. If such is not the case, why, for instance, in many large universities is instruction in the mathematics of the freshman year given to a majority of the students enrolled in these courses by assistants in the depart-

[1] Cf. article "A Study in Efficiency," *School Science and Mathematics*, October, 1910, pp. 579-81.